Quantum Nondemolition Measurement and Heralded Preparation of Fock States with Electromagnetically Induced Transparency in an Optical Cavity

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We propose a technique for quantum nondemolition (QND) measurement and heralded preparation of Fock states by dynamics of electromagnetically induced transparency (EIT). An atomic ensemble trapped in an optical cavity is driven by two external continuous-wave classical fields to form EIT in steady state. As soon as a weak coherent field is injected into the cavity, the EIT system departs from steady state, falls into transient state dynamics by the dispersive coupling between cavity injected photons and atoms. Because the imaginary part of time-dependent linear susceptibility $Im[X(t)]$ of the atomic medium explicitly depends on the number n of photons during the process of transient state dynamics, the measurement on the change of transmission of the probe field can be used for QND measurement of small photon number, and thus create the photon Fock states in particular single-photon state in a heralded way.

Introduction.—Quantum nondemolition (QND) measurement, which is designed to avoid the back action produced by a measurement [1], is of great importance in quantum information processing. For example, it provides the possible method to resolve small photon number states without changing them. Several QND measurement schemes have been proposed for monitoring the small photon number n [2–7]. In particular, Brune et at. [4] show that measurement of small photon number is possible through Rydberg atom phase-sensitive detection. Recently, this method [4] has been experimentally demonstrated to be used for progressive field-state collapse [8], quantum jumps of light recording the birth and death of a photon [9], and reconstruction of non-classical cavity field [10]. In Ref [4, 8–10], the photons are stored in a mode of a high quality microwave cavity and the QND measurement is made by the Ramsey interference of nonresonant Rydberg atoms that are sent through the cavity one by one. However, in the optical regime, QND measurement of photon number based on cavity quantum electrodynamics (CQED) is still a challenge.

On the other hand, Fock states exhibit no intensity fluctuations and a complete phase indetermination, which triggers off the research on the preparation and nonclassical nature of photon Fock states [8–11]. Single-photon Fock states in the optical regime, as the excellent information carrier, are desirable for applications in quantum information, e.g. unconditionally secure communication in quantum cryptography [12]. Many theories and experiments have been devoted to single-photon sources by accurately controlling a single emitter, such as individual atom [13], molecule [14], nitrogen vacancy center [15], or quantum dot [16]. In reality, the weak coherent state pulses, obtained by strongly attenuating a laser beam, are usually used for quantum key distribution (QKD). Those weak coherent state pulses have a Poissonian photon number statistics, and they present a large vacuum component, as well as non-negligible multiphoton contributions, which could make long-distance QKD suffer from a severe security loophole [17].

In this letter, we propose a technique for QND measurement and heralded preparation of photon Fock states in particular single-photon state, based on dynamics of electromagnetically induced transparency (EIT) in an optical cavity. EIT medium trapped inside a cavity has been explored for photon blockade [18], vacuum-induced transparency [19], photon-number selective group delay [20], NOON-state generation [21], single atoms EIT [22], and so on [23–25]. In our scheme, an atomic ensemble trapped in an optical cavity is driven by two external continuous-wave classical fields to form EIT in steady state. As soon as a weak coherent field enters the cavity, the EIT system departs from steady state, falls into transient state dynamics by the dispersive coupling between cavity injected photons and atoms. Because the imaginary part of time-dependent linear susceptibility $Im[X(t)]$ of the atomic medium explicitly depends on the number n of photons during the process of transient state dynamics, the change of transmission of the probe field can be used for QND measurement of the small photon number and heralded preparation of photon Fock states. The scheme proposed here has the following significant advantages: first, QND measurement in the optical regime is achieved through measurement of the change of transmission of the probe field in the dynamics of EIT, and the calculation shows QND measurement works in an cavity without strict strong coupling; second, the preparation of photon Fock states in particular single-photon state is probabilistic but in a heralded way.

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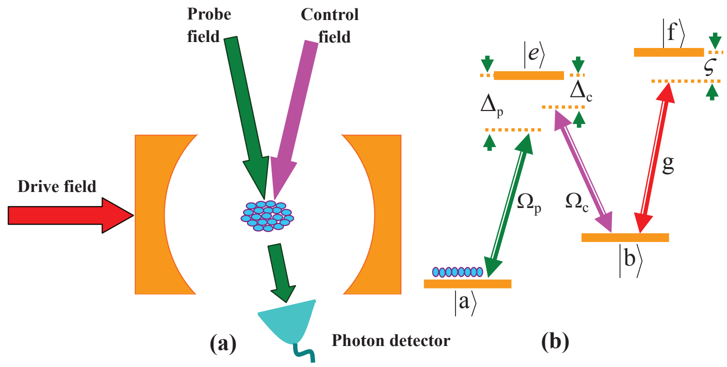

FIG. 1: (Color online) (a) Schematic setup to QND measurement and the preparation of photon Fock states based on dynamics of EIT. (b) The relevant atomic level structure and transitions.

Steady-state EIT in Λ-type configuration.—We first review the three-level EIT in steady state [26, 27]. As illustrated in Fig. 1, two continuous-wave classical fields, a probe field and a coupling field with central angular frequency ω_p and ω_c, respectively couple two lower metastable states $|a\rangle$ and $|b\rangle$ to upper level $|e\rangle$, and thus they form a standard Λ-type EIT configuration, in which the coherent processes are described by interaction Hamiltonian (within the rotating wave approximation)

$$H_1 = -(\Omega_p \sigma_{ea} + \Omega_c \sigma_{eb} + H.c.) + \Delta_p \sigma_{ee} + \delta \sigma_{bb}, \tag{1}$$

here, Ω_p (Ω_c) denotes the Rabi frequency associated with the probe field (the coupling field), $\sigma_{\xi\eta} = |\xi\rangle\langle\eta|$ ($\xi, \eta = a, b, e, f$) is the atomic projection operator, $\Delta_p = \omega_e - \omega_a - \omega_p$, $\Delta_c = \omega_e - \omega_b - \omega_c$ ($\hbar\omega_\xi$ is the energy of the level $|\xi\rangle$), and $\delta = \Delta_p - \Delta_c$. Under the two-photon resonant condition $\delta = 0$, and when the atomic ensemble has been driven into steady state, both real and imaginary parts of the linear susceptibility vanish in the ideal case, which means that the absorption and refraction of the probe field at the resonant frequency are eliminated. That leads to the transparency in otherwise absorbed medium [26, 27].

QND measurement with dynamics of EIT.— We consider such ensemble of atoms are trapped inside an optical cavity as shown in Fig. 1(a), the atoms couple to the quantized cavity field through the dipole coupling between level $|f\rangle$ and $|b\rangle$. Thus the interaction Hamiltonian is given by $H_i = \varsigma a^\dagger a + (g\sigma_{fb} a + H.c.)$, here $a(a^\dagger)$ is the annihilation (creation) operator of the cavity mode, g is the coupling constant between the cavity field and the atomic transition $|b\rangle \leftrightarrow |f\rangle$, and ς denotes the detuning of the cavity field from atomic transition $|b\rangle \leftrightarrow |f\rangle$. Suppose this coupling is in the case of large detuning, where the upper atomic level $|f\rangle$ can be adiabatically eliminated, the effective Hamiltonian is then given by $H_2 = G a^\dagger a \sigma_{bb}$, with the dispersive coupling strength $G = g^2/\varsigma$. We assume that under the two-photon resonant condition $\delta = 0$, the EIT system trapped in the cavity has been driven into steady state. Then there is a photon Fock state $|n\rangle$ in the cavity, the dynamics of the atom-photon density operator $\rho_{a-p}(t)$ is governed by the master equation

$$\begin{aligned}
\frac{d\rho_{a-p}(t)}{dt} &= -i[H_1 + H_2, \rho_{a-p}(t)] \\
&+ \frac{\gamma_{ea}}{2}[2\sigma_{ae}\rho_{a-p}(t)\sigma_{ea} - \sigma_{ee}\rho_{a-p}(t) - \rho_{a-p}(t)\sigma_{ee}] \\
&+ \frac{\gamma_{eb}}{2}[2\sigma_{be}\rho_{a-p}(t)\sigma_{eb} - \sigma_{ee}\rho_{a-p}(t) - \rho_{a-p}(t)\sigma_{ee}] \\
&+ \frac{\gamma_{deph}}{2}[2\sigma_{ee}\rho_{a-p}(t)\sigma_{ee} - \sigma_{ee}\rho_{a-p}(t) - \rho_{a-p}(t)\sigma_{ee}] \\
&+ \frac{\kappa}{2}[2a\rho_{a-p}(t)a^\dagger - a^\dagger a\rho_{a-p}(t) - \rho_{a-p}(t)a^\dagger a],
\end{aligned} \tag{2}$$

where γ_{ea} (γ_{eb}) is the spontaneous emission rate from state $|e\rangle$ to $|a\rangle$ ($|b\rangle$), γ_{deph} describes energy-conserving dephasing processes rates, and κ is the cavity decay rate. Equation (2) cannot be solved exactly. We numerically calculate the time-dependent linear susceptibility

$$X(t) = \frac{N\,|\mu_{ae}|^2\,\rho_{ea}}{\epsilon_0 \hbar \Omega_p}, \tag{3}$$

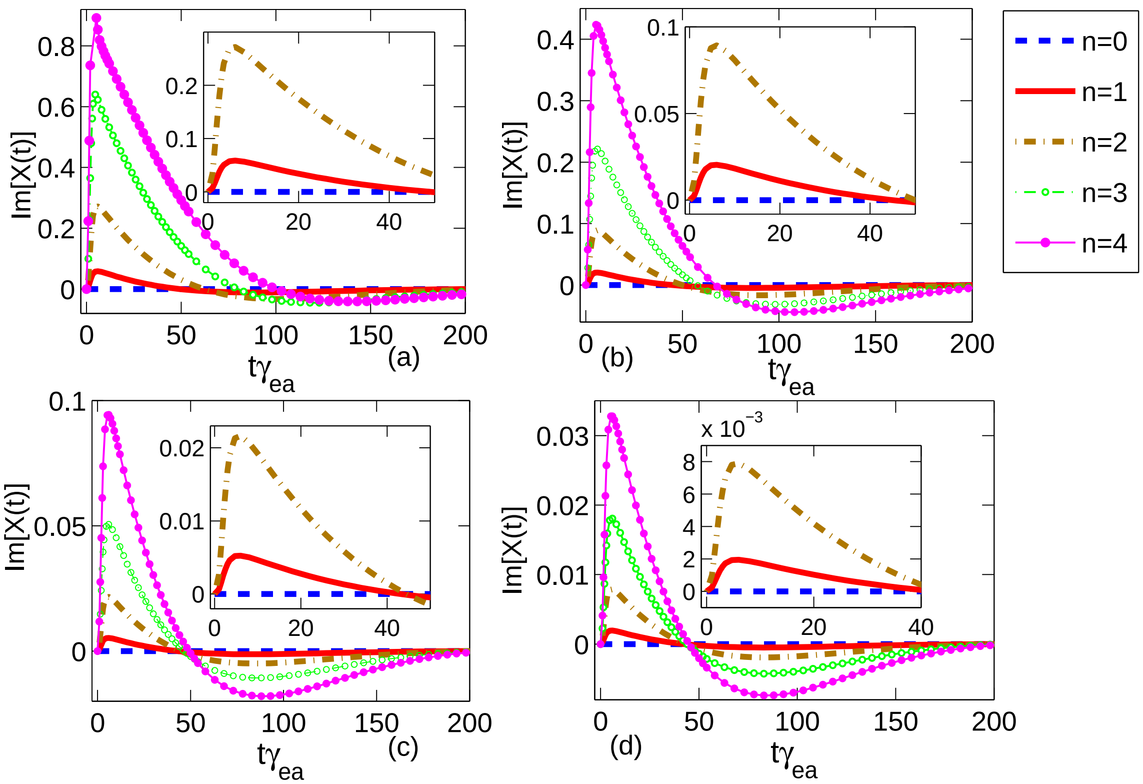

FIG. 2: (Color online) The imaginary part of time-dependent linear susceptibility $Im[X(t)]$ versus the time t in units of $1/\gamma_{ea}$, with different Fock state $|n\rangle$: $n=\{0,1,2,3,4\}$, when the dispersive coupling strength $G=$ (a) $0.08\gamma_{ea}$, (b) $0.03\gamma_{ea}$, (c) $0.008\gamma_{ea}$, (d) $0.003\gamma_{ea}$. Other common parameters are $\gamma_{eb}=\gamma_{ea}$, $\gamma_{deph}=0.1\gamma_{ea}$, $\kappa=0.3\gamma_{ea}$, $\Delta_p=\Delta_c=\Delta=-\gamma_{ea}$, $N\left|\mu_{ae}\right|^2/\gamma_{ea}\epsilon_0\hbar=1$, $\Omega_p=0.02\gamma_{ea}$, and $\Omega_c=0.2\gamma_{ea}$.

with the initial state $\rho_{a-p}(0)=\rho_a^{ste}\otimes|n\rangle$, where N is the atomic density, μ_{ea} is the dipole matrix element of the probe transition, ρ_{ea} denotes the off-diagonal density-matrix element, and ρ_a^{ste} is the atomic density operator under steady-state EIT. We give our attention to the imaginary part of linear susceptibility $Im[X(t)]$, which characterizes the absorption of atomic ensemble.

Figure 2 shows $Im[X(t)]$ versus the time t in units of $1/\gamma_{ea}$ for Fock state $|n\rangle$: $n=\{0,1,2,3,4\}$ with different dispersive coupling strength. From Fig.2, we see that the steady state EIT changes to the transient dynamics when the photons are injected into the cavity; furthermore, the peaks of $Im[X(t)]$ changes with the photon number n. Thus one could measure the peak transmission loss of the probe field to measure the photon number n. We can see the origin of this result, via the stochastic wave-function description of the dynamics of the system with non-Hermitian effective Hamiltonian [27] $H_{non}=-(\Omega_p\sigma_{ea}+\Omega_c\sigma_{eb}+H.c.)+a^{\dagger}a(G\sigma_{bb}-i\kappa/2)+(\Delta_p-i\gamma_{ea}/2)\sigma_{ee}+\delta\sigma_{bb}$. Under the two-photon resonant conditions $\delta=0$, the atomic system with the application of two classical fields moves into the dark state $|dark\rangle\propto\Omega_c|a\rangle-\Omega_p|b\rangle$, which has no population in excited state $|e\rangle$. Hence there are no spontaneous emission and no transmission loss of a probe field in the ideal case. Then there is a photon Fock state $|n\rangle$ $(n\neq 0)$, which shifts the level $|b\rangle$ and moves the atomic system out of the dark state $|dark\rangle$. Since the level shift is proportional to photon number n, the dynamics of EIT departing from dark state $|dark\rangle$ is explicitly dependent on the photon number n.

We can enhance the signal-to-noise ratio for the measurement process involving accumulated measurement. In Fig.3, we plot the area

$$S=\int_0^T Im[X(t)]dt, \tag{4}$$

with the different dispersive coupling strength G: $G=\{0.08\gamma_{ea},0.03\gamma_{ea},0.008\gamma_{ea},0.003\gamma_{ea}\}$ when $T=50/\gamma_{ea}$. We find that the area S strongly depends on the photon number n even if $G<\kappa,\gamma_{ea}$. The photons are dispersively coupled to the atoms, and when the photon number is small, $g\sim 10G$. Thus the cooperativity parameter $\eta=g^2/\kappa\gamma_{ea}$

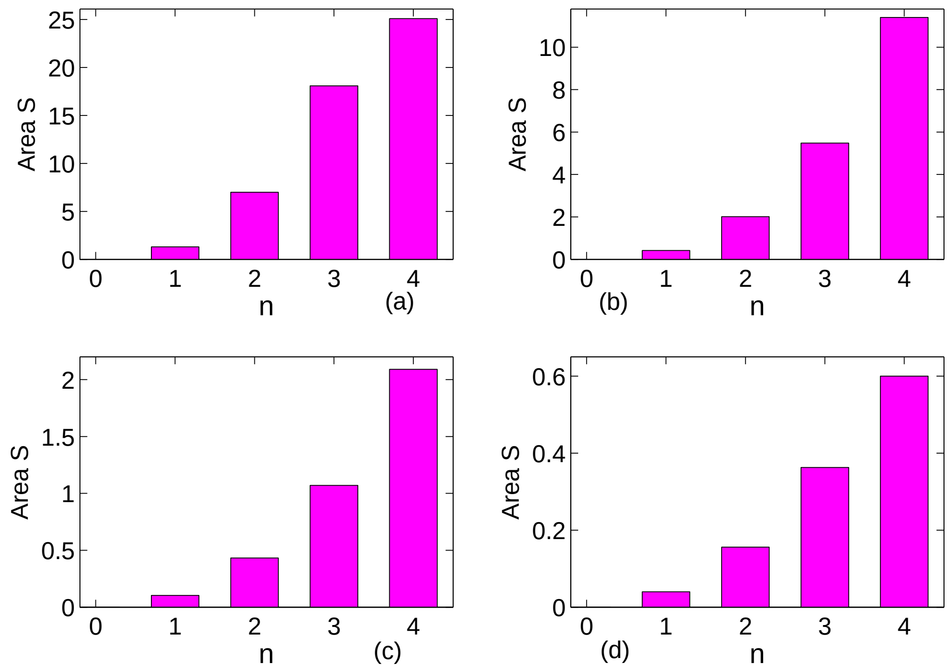

FIG. 3: (Color online) The area $S = \int_0^T Im[X(t)]dt$ $(T = 50/\gamma_{ea})$ with the different dispersive coupling strengh $G =$ (a) $0.08\gamma_{ea}$, (b) $0.03\gamma_{ea}$, (c) $0.008\gamma_{ea}$, (d) $0.003\gamma_{ea}$. Other parameters are the same as in Fig. 2.

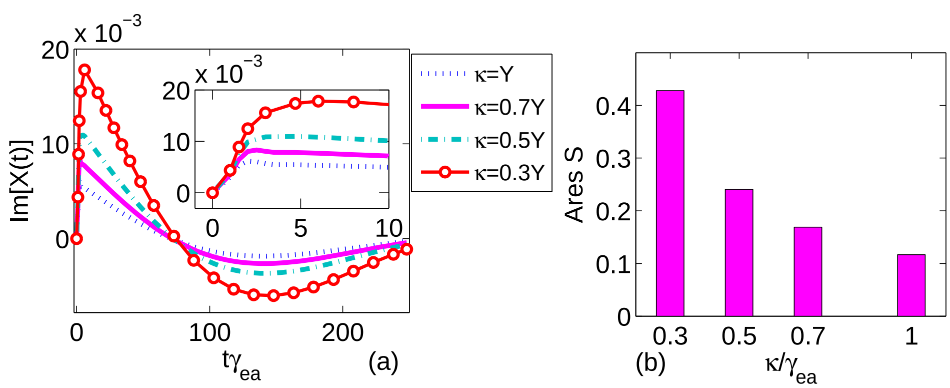

FIG. 4: (Color online)(a) The imaginary part of time-dependent linear susceptibility $Im[X(t)]$ for the single-photon Fock state $|1\rangle$, with the different cavity decay rate κ: $\kappa = \{\gamma_{ea}, 0.7\gamma_{ea}, 0.5\gamma_{ea}, 0.3\gamma_{ea}\}$. (b) the area $S = \int_0^T Im[X(t)]dt$ $(T = 50/\gamma_{ea})$ in (a). The parameter $G = 0.03\gamma_{ea}$ and other parameters are the same as in Fig. 2.

of the optical cavity in Fig. 3 (a), (b), (c), and (d) are $\eta_a \sim 2$, $\eta_b \sim 0.3$, $\eta_c \sim 0.02$, and $\eta_d \sim 0.003$, respectively. Figure 3 indicates there are evident changes of transmission of a probe field with different Fock state $|n\rangle$, even if the cooperativity parameter $\eta < 1$, which means that a measurement on the changes of transmission of the probe field can be used for effective QND measurement of the small photon number, even if the optical cavity is not in strong coupling regime.

Next we explore the influences of cavity decay rate κ and the single-photon detuning $\Delta_p = \Delta_c = \Delta$ for the single-photon Fock state $|1\rangle$. In Fig.4, we investigate the curve $Im[X(t)]$ and the area S , with the different cavity decay rate κ: $\kappa = \{\gamma_{ea}, 0.7\gamma_{ea}, 0.5\gamma_{ea}, 0.3\gamma_{ea}\}$. Figure 4 indicates that there is a visible change of transmission of a probe field, even when $\kappa = \gamma_{ea}$ and the injected photon is in single-photon Fock state $|1\rangle$. Figure 5 shows the

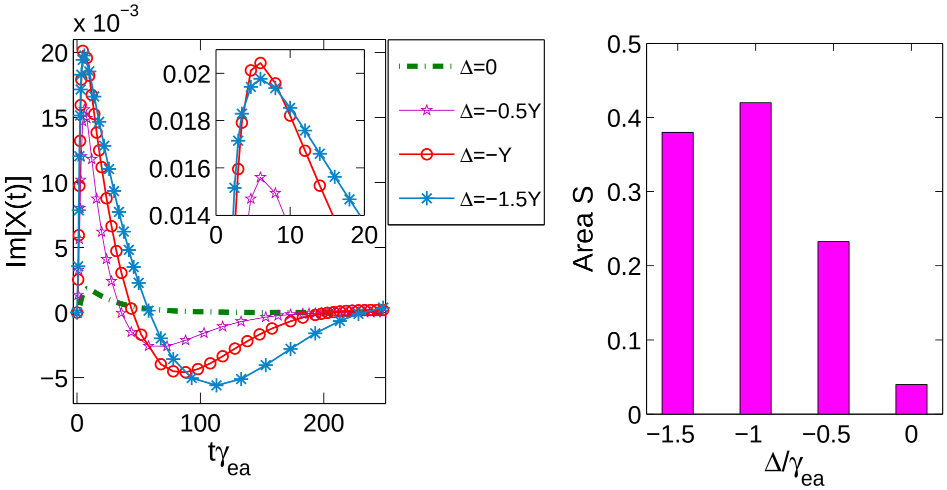

FIG. 5: (Color online)(a) The imaginary part of time-dependent linear susceptibility $Im[X(t)]$ for the single-photon Fock state $|1\rangle$, with the single-photon detunings $\Delta_p = \Delta_c = \Delta$: $\Delta = \{0, -0.5\gamma_{ea}, -\gamma_{ea}, -1.5\gamma_{ea}\}$. (b) the area $S = \int_0^T Im[X(t)]dt$ $(T = 50/\gamma_{ea})$ in (a). The parameter $G = 0.03\gamma_{ea}$ and other parameters are the same as in Fig. 2.

curve $Im[X(t)]$ and the area S for the single-photon Fock state $|1\rangle$, with the single-photon detunings $\Delta_p = \Delta_c = \Delta$: $\Delta = \{0, -0.5\gamma_{ea}, -\gamma_{ea}, -1.5\gamma_{ea}\}$. From Fig.5, we see that an appropriate single-photon detuning, e.g. $\Delta = -\gamma_{ea}$, may effectively increase the total changes of the transmission of the probe field.

We note that Reference [19] has shown that the average cavity photon number $\langle n_c\rangle$ can be determined by a measurement on the peak transparency of the probe field. However, our protocol is much different from that in Ref. [19], in which the average cavity photon number $\langle n_c\rangle$ is measured and quantum state of cavity field changes during the measurement even if it is initially in a certain Fock state.

Heralded photon Fock states source with QND measurement.—In the following part, we consider the pulsed excitation of the cavity. The interaction Hamiltonian for the cavity mode a driven by a laser field is [18] $H_{dri} = \sqrt{2\kappa}(\beta^* a - \beta a^\dagger)$, where β is the laser field amplitude. When $\beta \gg \sqrt{\kappa}$, a weak coherent field $|\alpha\rangle$ can be generated within a short time, in which the cavity decay can be neglected. Assuming that a weak coherent field is injected into the cavity, an observable photon number n is measured by detecting the change of transmission of the probe field, which makes the coherent field collapse into a certain Fock state. Thus a probable, but heralded small photon Fock states in particular single-photon state is created. After the time $t' > 200/\gamma_{ea}$, the transient dynamics of EIT returns to steady state EIT, and if another weak coherent field is injected into the cavity, the system repeats the above process. Thus many weak coherent fields are sequently injected into the cavity at time $t' > 200/\gamma_{ea}$ interval, and detection of the change of transmission loss of the probe field will realize a heralded single-photon source, which well suits for actual QKD [12].

Discussion and conclusion.— The transmitted intensity of a probe beam that has the wavelength λ can be obtained from the relation $I_p = I_0 e^{-\alpha l}$, here $\alpha = 2\pi Im[X(t)]/\lambda$ is the absorption coefficient, l is the length of a cold atomic sample, and I_0 is incident intensity of the probe beam. Thus the transmission change of a classical optical field is $P = \int_0^T (I_p - I_0)/I_0 dt = \int_0^T (1 - e^{-\alpha l})dt \sim \int_0^T \alpha l dt = 2\pi S l/\lambda$ (when $\alpha l \ll 1$). Because the area S strongly depends on the photon number n, QND measurement can be achieved by the measurement of the transmission change of a classical optical field. The measurement time $T = 50/\gamma_{ea} \gg 1/\kappa$, so this means that our scheme does not require that the measurement time is shorter than the cavity decay time $1/\kappa$. There is a visible change of transmission loss of a probe field, even when the injected photon is in single-photon Fock state $|1\rangle$. Thus our scheme provides a heralded single-photon source for actual QKD [12]. In our scheme, the numerical calculation shows our scheme still works even if the cooperativity parameter $\eta < 1$, and this may largely lower the experimental requirements.

Now we address the experiment feasibility of the proposed scheme. We consider an ensemble of ^{87}Rb atoms trapped in an optical cavity. The states $|a\rangle$ and $|b\rangle$ correspond to $|F = 1, m = -1\rangle$ and $|F = 1, m = 1\rangle$ of $5S_{1/2}$ ground levels respectively, while $|e\rangle$ and $|f\rangle$ correspond to $|F = 1, m = 0\rangle$ and $|F = 1, m = 1\rangle$ of $5P_{3/2}$ excited level, respectively. The relevant cavity QED parameters in the experiment are $(g_0, \kappa, \gamma_e)/2\pi = (27, 4.8, 6)MHz$ [28], which correspond to the cooperativity parameter $\eta = g_0^2/\kappa\gamma_e = 25 \gg 1$. Under the experimental conditions, the principle experiment

could be realized.

In summary, we have proposed a method for QND measurement and heralded preparation of Fock states with dynamics of EIT in an optical cavity. In our scheme, an atomic medium trapped in an optical cavity driven by two continuous-wave classical fields moves into a dark state. Then a weak coherent field enters the cavity, shifts the level $|b\rangle$ and moves the atomic medium out of the dark state. Because the curve of $Im[X(t)]$ explicitly depends on the number n of photons, a measurement on the changes of transmission of the probe field can be used for effective QND measurement of the small photon number and preparation of Fock states. Our numerical calculation shows the scheme still works in an optical cavity without strong coupling. Thus our method opens an alternate for QND measurement and preparation of photon Fock states in particular single-photon state in a heralded way.

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